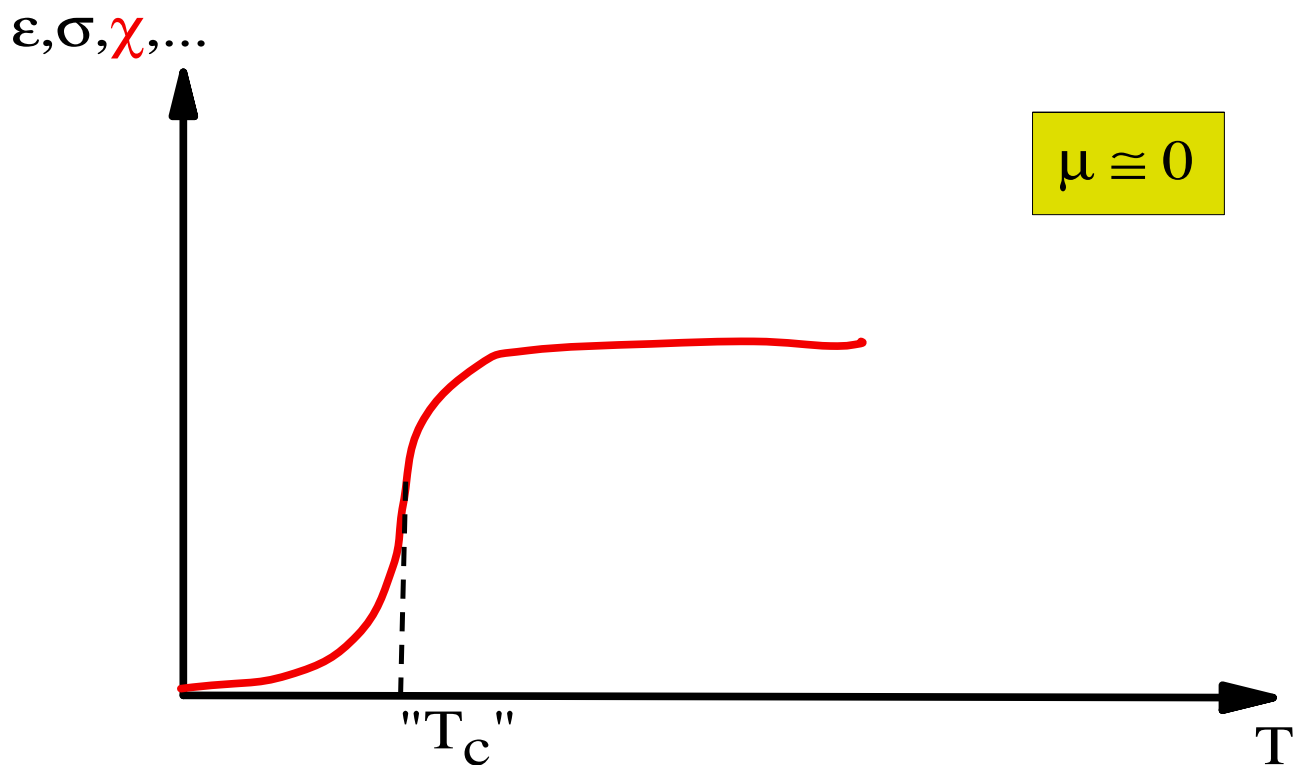
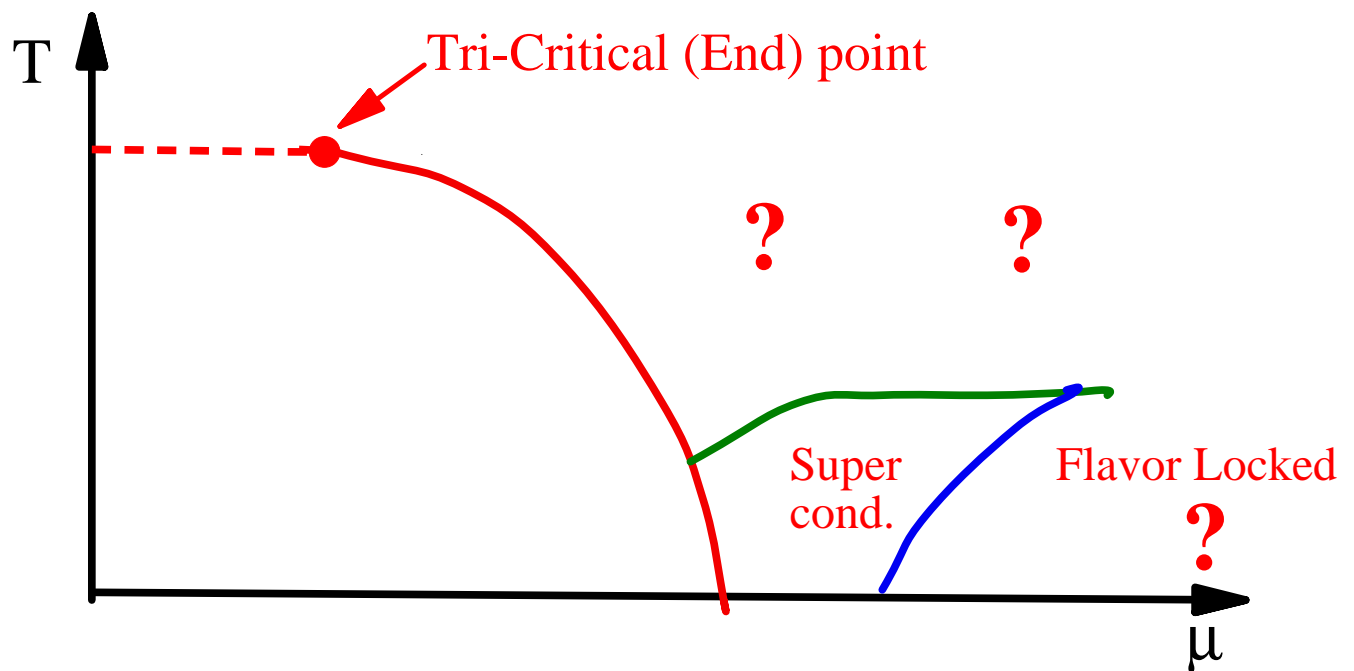


Event by Event fluctuations and the QGP

- Introduction and general discussion
- What can be addressed with E-by-E fluctuations
- Fluctuations of particle ratios
 - **Charge fluctuations!**
 - Fluctuations of conserved quantities
- Outlook

The QCD Phase-Diagram



Event-by-event fluctuations

The old idea: Two distinct event classes



After the first experiments (NA49): GAUSSIANS, GAUSSIANS....

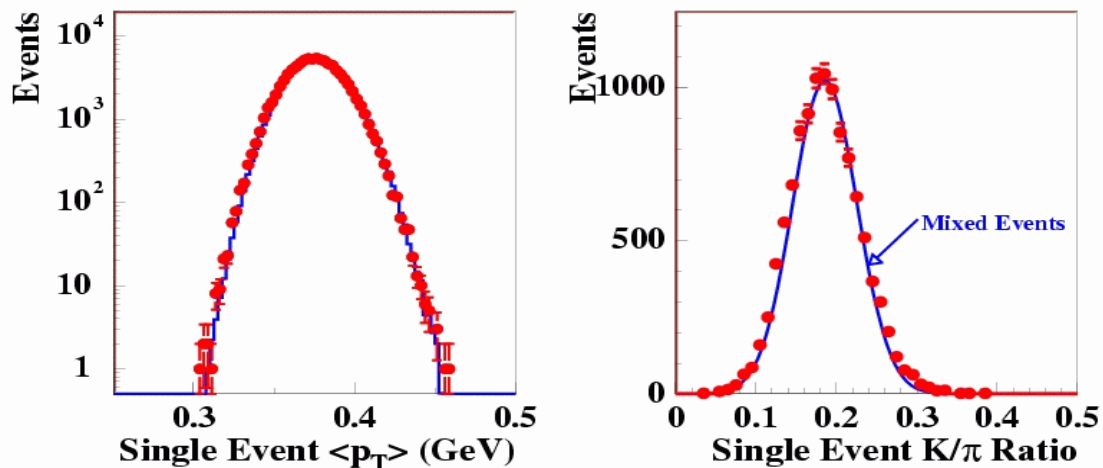


Physics is in the width of the Gaussian:

Event-by-Event

- First possibilities with NA49 and STAR
- first data from NA49
 - ⇒ Gaussians, with almost ($\pm 2\%$) statistical width!
- Theory is just now developing

NA49 Pb+Pb Event-by-Event Fluctuations

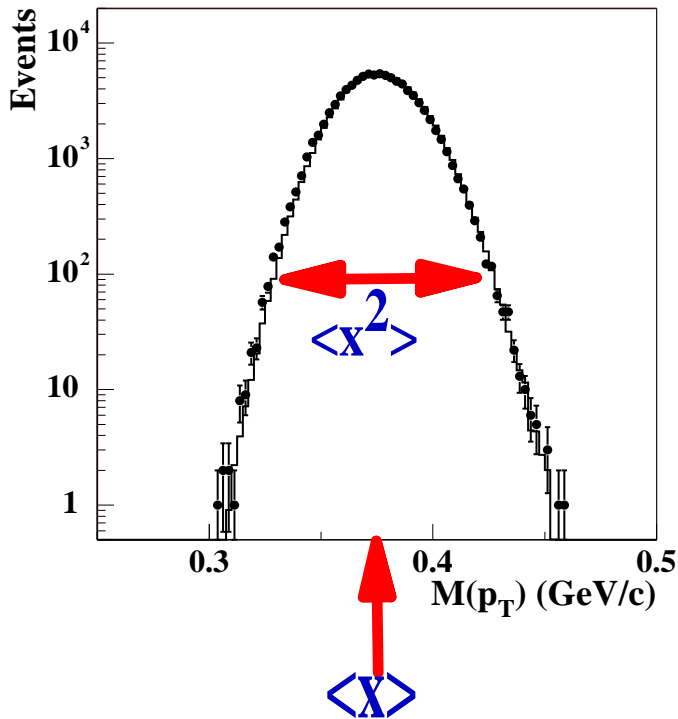


Dynamical Event-by-Event Fluctuations: $< 1\%$ in $\langle p_T \rangle$
 $< 15\%$ in K/π

E-by-E activities

- Temperature fluctuations (Stodolsky (95), Shuryak (98))
- "Phi"-measure (Gazdzicki,Mrowczynski(92))
- Quantum statistics (Mrowczynski (98))
- Two-particle correlations (Bialas, VK(99),Belkacem et al (99))
- Particle Ratios (Baym,Heiselberg(99), Jeon, VK (99,00))
- Resonance gas (Rajagopal,Shuryak,Stephanov (99), Jeon,VK (99))
- Phase transitions, bubble formation (Baym,Heiselberg (99), Rajagopal, Shuryak, Stephanov (99),Heiselberg, Jackson(00))
- charge fluctuations (Asakawa,Heinz Mueller(00), Jeon,VK(00),Dumitru,Pisarski(00), Stephanov,Shuryak(00))
- balance functions (Bass, Danielewicz,Pratt (00))
- Baryon number fluctuations(Asakawa,Heinz,Mueller(00), Gavin(00))
- Review article: Heiselberg, nucl-th/000304
-

Gaussians it is!



The physics is in
the width!

For Gaussian shape, E-by-E measures 2-particle-correlations
(A. Bialas, VK, PLB 456 (99) 1)

⇒ Two arm spectrometer is sufficient

requires event-by-event

two-arm spectrometer is enough



Fluctuations (Widths)

measure generalized susceptibilities

$$\chi_{a,b} = \frac{dF}{da db}$$

Examples:

- Heat capacity \Leftrightarrow T (or p_t) - fluctuations
- Baryon number susceptibility
 \Leftrightarrow baryon number fluctuations
- charge susceptibility
 \Leftrightarrow charge fluctuations

These susceptibilities can be calculated by
Lattice QCD

Volume fluctuations

Even for the tightest trigger conditions, the volume created in and HI-collision **fluctuates** !

Typical observable:

$$\hat{O} = \rho V$$

Thus the fluctuations of O are "contaminated" by **volume fluctuations**

$$\delta \hat{O} = (\delta \rho) V + \rho \delta V$$

The physics is in **($\delta \rho$)** !!!!


⇒ Study intensive (**volume independent**) quantities

- Particle Ratios ⇐ **this talk**
- mean transverse momentum
- ...

Fluctuations of Particle Ratios

$$R = \frac{N_1}{N_2}$$

Correlations!!!

$$\begin{aligned} \frac{\langle (\delta R)^2 \rangle}{\langle R \rangle^2} &= \left\langle \left(\frac{\delta N_1}{\langle N_1 \rangle} - \frac{\delta N_2}{\langle N_2 \rangle} \right)^2 \right\rangle \\ &= \left(\frac{\langle (\delta N_1)^2 \rangle}{\langle N_1 \rangle^2} + \frac{\langle (\delta N_2)^2 \rangle}{\langle N_2 \rangle^2} - 2 \frac{\langle \delta N_1 \delta N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle} \right) \\ &= \left(\frac{\omega_1}{\langle N_1 \rangle} + \frac{\omega_2}{\langle N_2 \rangle} - 2 \frac{\langle \delta N_1 \delta N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle} \right) \end{aligned}$$


$$\langle (\delta N)^2 \rangle = \omega N$$

For poisson statistics (ideal gas): $\omega = 1$

Bose statistics:	$\omega > 1$	} small corrections
Fermions	$\omega < 1$	

NO Volume Fluctuations !!!

$$\begin{aligned}\frac{\delta N_1}{\langle N_1 \rangle} - \frac{\delta N_2}{\langle N_2 \rangle} &= \frac{\delta \rho_1 V + \rho_1 \delta V}{\rho_1 V} - \frac{\delta \rho_2 V + \rho_2 \delta V}{\rho_2 V} \\ &= \frac{\delta \rho_1}{\rho_1} - \frac{\delta \rho_2}{\rho_2}\end{aligned}$$

For $N_2 \ll N_1$ (such as K/π ratio)

$$\frac{\langle (\delta R)^2 \rangle}{\langle R \rangle^2} \cong \frac{\omega_2}{\langle N_2 \rangle}$$

\Rightarrow "simple" statistics dominated by the fluctuations of the smaller number

\Rightarrow NA49 results for K/π ratio!

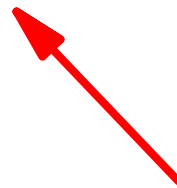
Look at $N_1 \approx N_2$ in order to find correlations

Fluctuations of π^+/π^-

(with S. Jeon, PRL83 (1999) 5435)

Consider fluctuations of $R = \frac{\pi^+}{\pi^-}$

$$\frac{\langle (\delta R)^2 \rangle}{\langle R^2 \rangle} = \left(\frac{\langle (\delta \pi^+)^2 \rangle}{\langle \pi^+ \rangle^2} + \frac{\langle (\delta \pi^-)^2 \rangle}{\langle \pi^- \rangle^2} - 2 \frac{\langle \pi^+ \pi^- \rangle - \langle \pi^+ \rangle \langle \pi^- \rangle}{\langle \pi^+ \rangle \langle \pi^- \rangle} \right)$$



Correlation term

- No contribution from volume fluctuations
- $\rho^0 \rightarrow \pi^+ \pi^-$, $\omega \rightarrow \pi^+ \pi^- \pi^0$ reduce fluctuations

Sensitive to number of rho and omega at hadronization!!!!

Predict: For thermal weights fluctuations should be only 70 % of statistical.

Find: Deviations from statistical fluctuation are very sensitive to rho and omega at *hadronization*
 \Leftrightarrow Dilepton Production

Fluctuations of H^+ / H^-

(with S. Jeon, PRL85 (00) 2076)

Consider: $R = \frac{N^+}{N^-} = \frac{1+F}{1-F}$ $F = \frac{N^+ - N^-}{N^+ + N^-} = \frac{Q}{N_{ch}}$

$Q = N_+ - N_-$ $N_{ch} = N_+ + N_-$
 Net charge

$$\langle (\delta R)^2 \rangle = 4 \langle (\delta F)^2 \rangle \cong 4 \frac{\langle (\delta Q)^2 \rangle}{\langle N_{ch} \rangle^2}$$

Propose Observable:

$$\langle N_{ch} \rangle \langle (\delta R)^2 \rangle \cong 4 \frac{\langle (\delta Q)^2 \rangle}{\langle N_{ch} \rangle} \propto \frac{\chi}{\sigma}$$

$$\left. \begin{array}{l} \chi = -\frac{\partial^2 \phi}{\partial \mu_Q^2} \quad \text{Charge-Susceptibility} \\ \sigma \quad \quad \quad \text{Entropy-density} \end{array} \right\} \text{Lattice}$$

Fluctuations in H^+ / H^-

Quark Gluon Plasma:

$$\frac{\langle (\delta Q)^2 \rangle}{\langle N_{ch} \rangle} \propto \frac{\langle (\delta Q)^2 \rangle}{S} \frac{S}{\langle N_{ch} \rangle} = \frac{Q_u^2 N_u + Q_d^2 N_d}{4(N_u + N_d + N_g)} \frac{S}{\langle N_{ch} \rangle}$$

$$N_{ch} \langle (\delta R)^2 \rangle \approx 1 - 1.4$$

Pion Gas:

$$\frac{\langle (\delta Q)^2 \rangle}{\langle N_{ch} \rangle} = \frac{1^2 \langle N_{\pi^-} \rangle + 1^2 \langle N_{\pi^+} \rangle}{(\langle N_{\pi^+} \rangle + \langle N_{\pi^-} \rangle)}$$

$$\langle N_{ch} \rangle \langle (\delta R)^2 \rangle = 4$$

Hadron gas, incl. resonances (~30% reduction of $\langle (\delta R)^2 \rangle$)

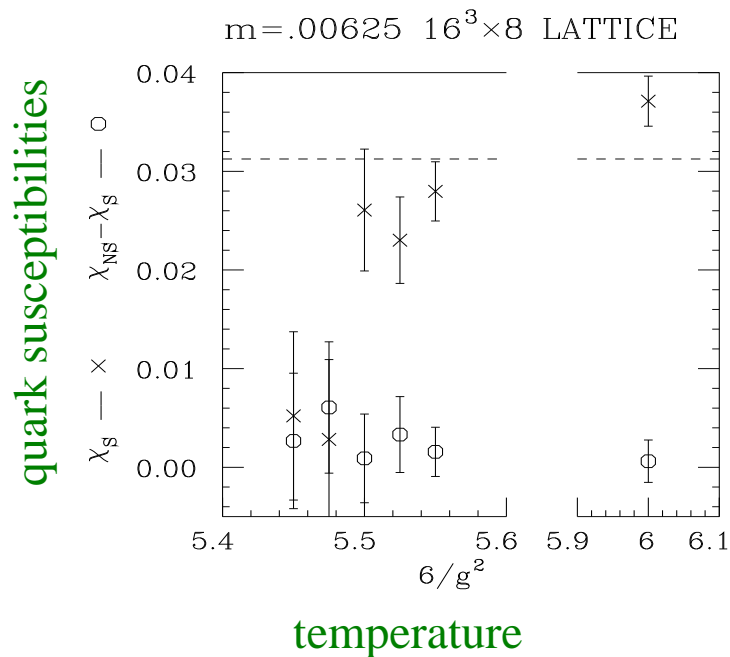
$$\langle N_{ch} \rangle \langle (\delta R)^2 \rangle \cong 3$$

Observable changes by **Factor ~ 2-3** if phase transition

Reminder of: $R_{e^+e^-} \equiv \frac{e^+e^- \rightarrow \text{Hadrons}}{e^+e^- \rightarrow \mu^+\mu^-} = N_c \sum_q Q_q^2$

Lattice Results

(Gottlieb et al, PRD55(97) 6852)



Findings: $\langle \delta N_u \delta N_d \rangle \approx 0;$

$$\langle \delta N_f^2 \rangle \approx \langle N_f \rangle$$

$$\langle S_{\text{gluon}} \rangle \approx 3.6 \langle N_{\text{gluon}} \rangle$$

$$\langle S_{\text{quarks}} \rangle \approx (1/2) \times 4.2 \langle N_{\text{quarks}} \rangle$$

Simple estimate for $(\langle \delta Q^2 \rangle / S)$ agrees well with present Lattice results

Fluctuations of conserved quantities

Asakawa, Heinz, Mueller, PRL85 (00) 2072

S. Jeon and V.K. PRL85 (00) 2076

General idea: Given strong longitudinal expansion the fluctuations of **conserved** quantities will be preserved during hadronization and hadronic phase

Requirements: Equilibration and buildup of longitudinal flow in partonic phase

This is **NOT** an **adiabatic** transition through T_c

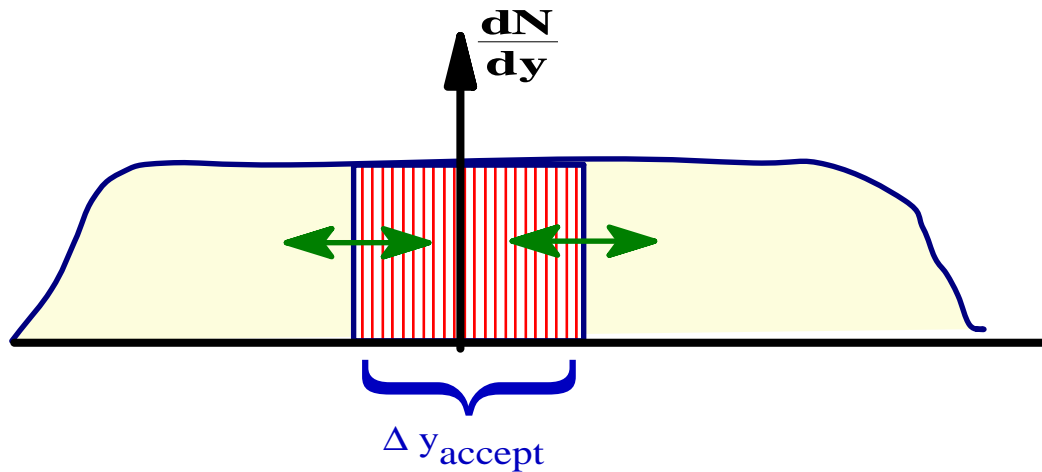
Examples:

- charge (**this talk**)
- baryon number (**neutrons??**)
- strangeness

Caution: These are **extensive** quantities
 \Rightarrow need to construct **intensive** quantity
such as N^+/N^- - ratio

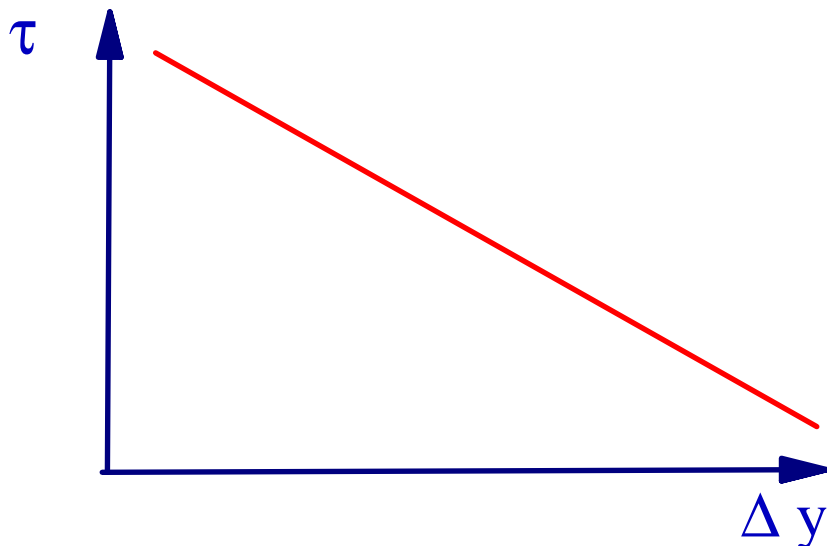
Rescattering

(hadronization)

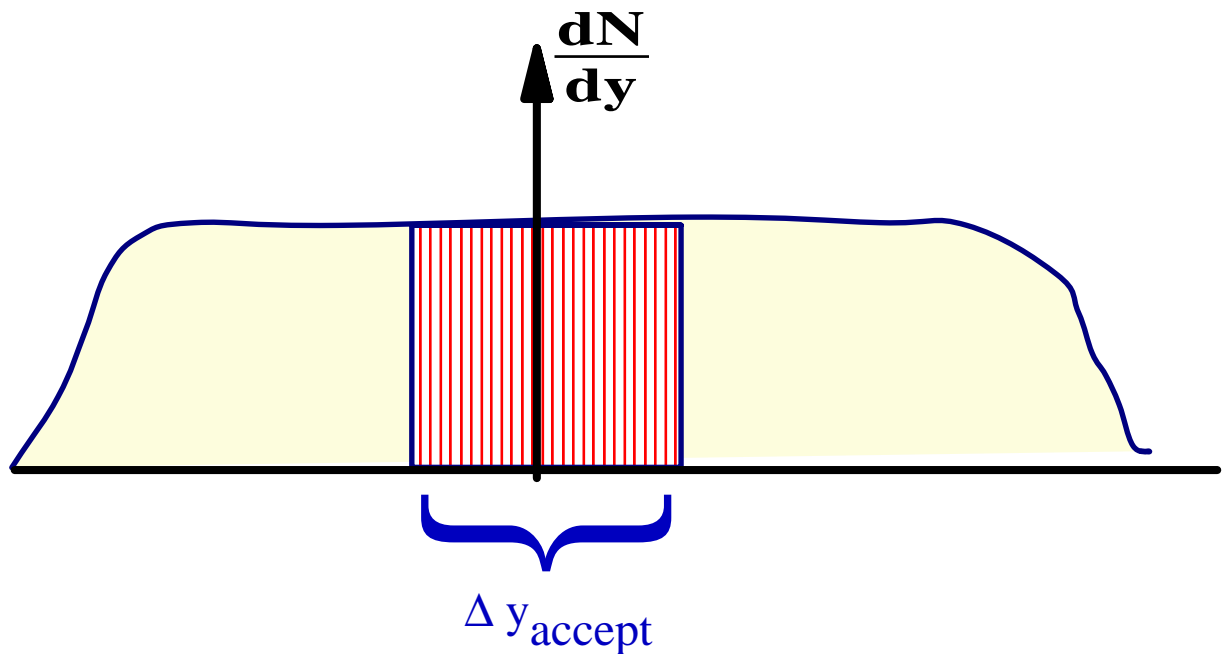


If: $\Delta y_{\text{accept}} \gg \Delta y_{\text{collision}} \Rightarrow$ negligible

General trend: The **larger** Δy the **deeper** one looks into the event



Acceptance



For $\Delta y_{\text{accept}} \ll \Delta y_{\text{total}} \Rightarrow$ grand canonical ensemble for $\langle \delta Q^2 \rangle$

Correction for charge conservation: $F_Q = (1 - \frac{\langle N_{ch} \rangle_{\Delta y}}{\langle N_{ch} \rangle_{\text{total}}})$

In Bjorken picture:

Within Δy_{accept} $\left\{ \begin{array}{l} \text{charge is conserved} \\ S_{\text{initial}} \leq S_{\text{final}} \end{array} \right.$

\Rightarrow

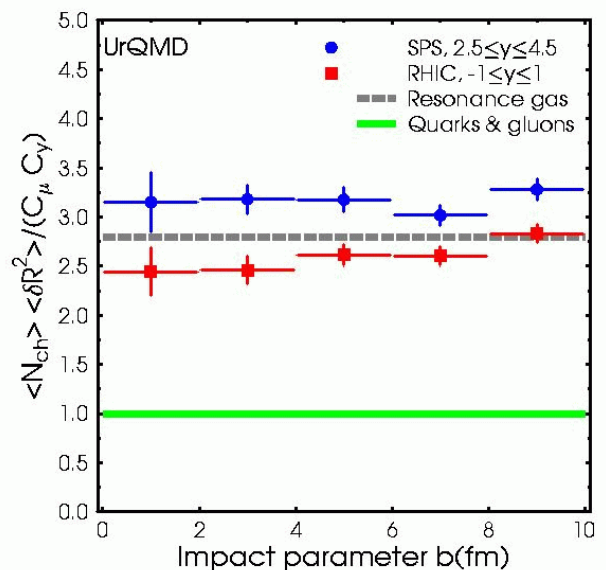
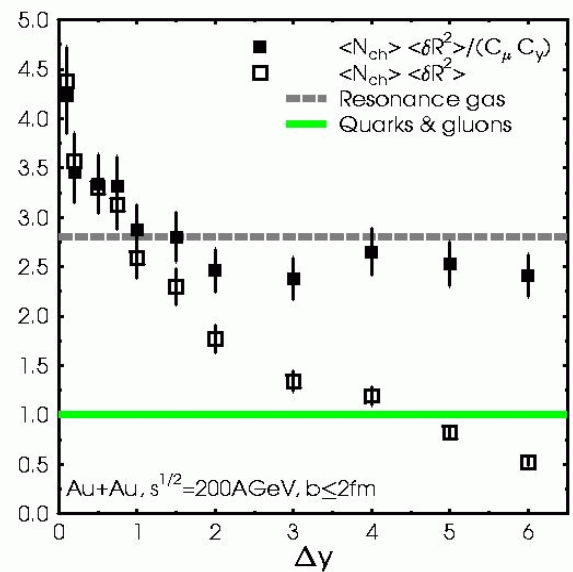
$$\left. \frac{\langle (\delta Q)^2 \rangle}{S} \right|_{\text{final}} \leq \left. \frac{\langle (\delta Q)^2 \rangle}{S} \right|_{\text{initial}}$$

Transport results

(URQMD)

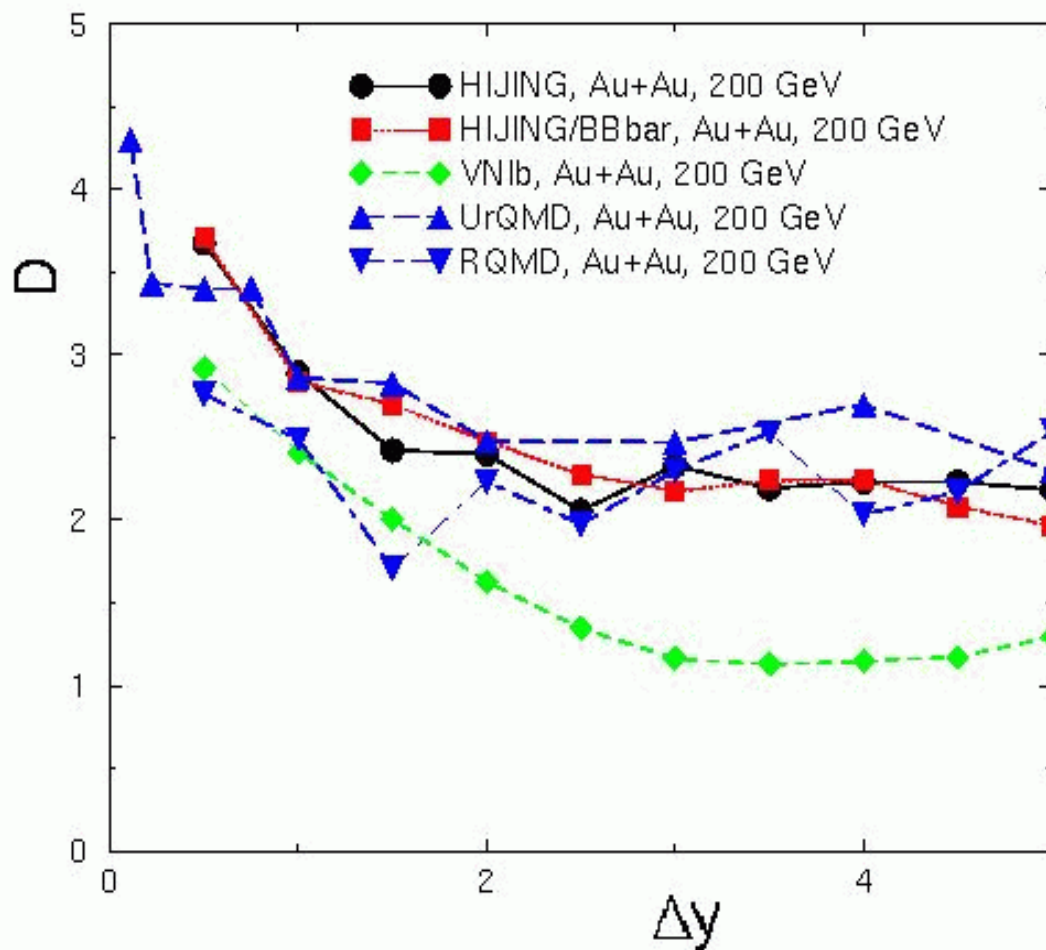
(M. Bleicher, S. Jeon and V.K. PRC62 (00) 061902)

- Acceptance corrections essential
- works for wide range of b and y
- URQMD consistent with prediction for Hadron Gas
- $\Delta y > 1$ to capture resonances



Transport results Comparison

(C.Gale, V. Topor Pop and Q.H. Zhang)



Caveats

- Hadronization (Parton cascade....)
- Resonances? (NO!)
- Finite acceptance corrections (solved)
- Rescattering in hadronic phase (not an issue, choose Δy sufficiently large \Rightarrow LHC)
- Mixture of QGP and HG (always a problem)
- proton-proton (same as HG, lots of data in the '70)
-

Rescattering

(some numbers)

More detailed discussion based on diffusion eq:
(Shuryak and Stephanov, hep-ph/0010100)

$$\langle (\delta Q)^2 \rangle = \langle (\delta Q)^2 \rangle_{equil.} F(x) + \langle (\delta Q)^2 \rangle_0 (1 - F(x))$$

$$x = \frac{\Delta y_{diff}}{(\Delta y_{accept}/2)} \quad (\Delta y_{diff})^2 = \int_0^{\tau} (y_{coll})^2 \frac{d\tau}{\tau_{free}}$$

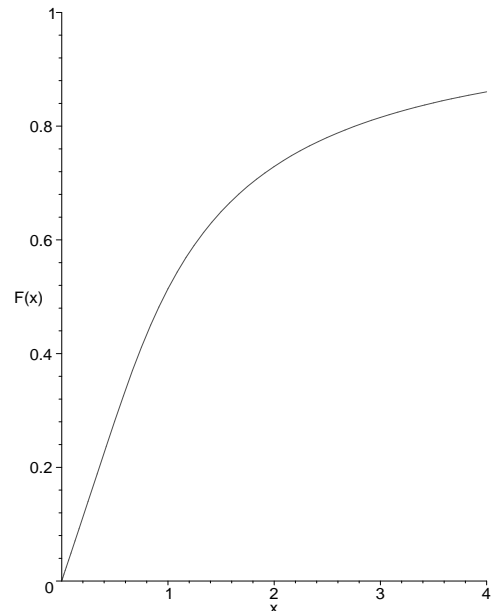
Hadronic transport ($\Delta y=4$):

$$y_{coll} \approx 0.8, \quad \Delta y_{diff} \approx 0.6$$

$$\Rightarrow x \approx 0.6$$

$$\Rightarrow F(x) = 0.34$$

$$\frac{\langle (\delta Q)^2 \rangle}{\langle (\delta Q)^2 \rangle_{equil.}} = 0.65$$



30% correction

Experimental issues

Detectors do fluctuate as well!

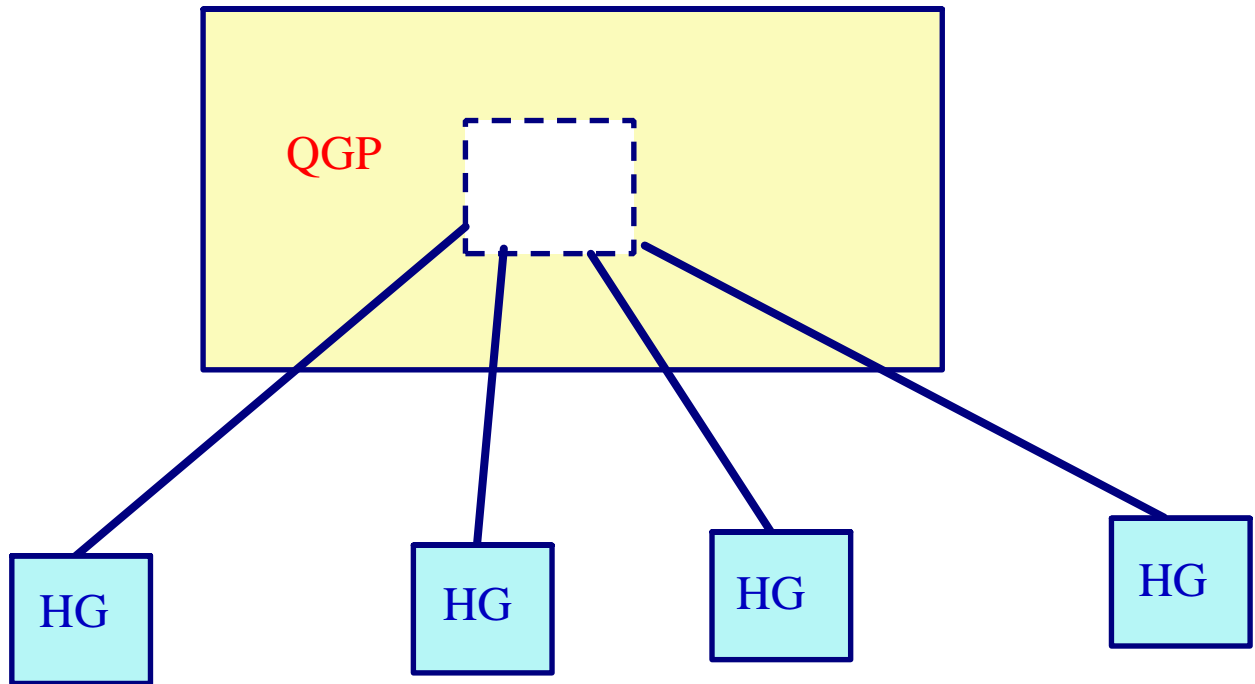
$$\langle \delta Q^2 \rangle_{\text{measured}} = \langle \delta Q^2 \rangle_{\text{real}} + \Delta^2(\text{detector})$$

example: $N = 1000$

$$\Rightarrow \langle \delta N^2 \rangle / N \approx 3\%$$

efficiency fluctuations

Speculate



Problem: Ensemble of HG-boxes should carry the same fluctuations per (entropy/energy) as QGP.
THIS is not a HG in chemical equilibrium

One way out (which at least does not violate obvious symmetries):

Enhance $I=0$ mesons, such as omega, eta ...

Simple estimate: ω up by factor $\sim 5 \Rightarrow$ PHENIX

Conclusions

- E-by-E measures particle correlations!
- Look at intensive variables to avoid Volume fluctuations
- Ratio fluctuations! (*NOT* Ratios of Fluctuations)
- π^+/π^- : chemical equilibrium
- N^+/N^- : QGP-test
 - $\langle N_{\text{ch}} \rangle \langle (\delta R)^2 \rangle \approx 3$ for hadron gas
 - $\langle N_{\text{ch}} \rangle \langle (\delta R)^2 \rangle \approx 1$ for QGP
- Other applications: such as
 - K^+/K^- etc....
 - tri-critical point (lots of σ -mesons)
 - multifragmentation
 - ...
- Waiting for the Data....

Absence of evidence is not evidence of absence

Conclusions (contd.)

